

## Appendix A

are regarded as indices over the same universe  $U_a$ . Either interpretation is legitimate; in the latter case, however, the observer is explicitly talking about relation  $R \subset X \times Y \in U_s$  which may be due to undetermined or other than deterministic connections. Thus, either by juxtaposing two distinct universes  $U_a$ ,  $U_b$  or by considering  $X$  and  $Y$  as alternative sets over the same universe, it is possible to assign probability numbers to joint events designated by pairs  $\langle x, y \rangle \in X \times Y$ ; namely, the joint event probabilities  $p(x, y)$ . The joint uncertainty/information index, relevant to the selection of  $\langle x, y \rangle$  in  $U_a$  is

$$H_a(x, y) = -\sum_x \sum_y p(x, y) \log p(x, y)$$

and if  $x, y$  index separate universes  $U_a$ ,  $U_b$  then this is identical with  $H(\alpha, \beta)$ . Any correlation between occurrences indexed by  $x$  and occurrences indexed by  $y$  reduces the value of this quantity. Its maximum value occurs in the absence of correlation and is

$$\begin{aligned} H^*(x, y) &= H_a(x) + H_a(y) \\ \text{or } H^*(\alpha, \beta) &= H(\alpha) + H(\beta) = H^*(U_a \cup U_b) \end{aligned}$$

The degree of coupling is obtained as a difference.

$$T_a(x, y) = H^*(x, y) - H_a(x, y) = H_a(x) + H_a(y) - H_a(x, y)$$

or

$$T(\alpha, \beta) = H(\alpha) + H(\beta) - H(\alpha, \beta) \quad (2)$$

(3) The former interpretation,  $T_a(x, y)$ , is best regarded as a *coupling* and the latter,  $T(\alpha, \beta)$ , as a *transmission*. Though the forms are identical, 'coupling' exists between features of a *system*; regarded on other grounds, as coherent or unitary. 'Transmission' takes place between systems regarded, on other grounds, as distinct. In either case, the term in question is positive or zero.

$$\begin{aligned} H^*(x, y) &\geq T_a(x, y) \geq 0 \\ H^*(\alpha, \beta) &\geq T(\alpha, \beta) \geq 0 \end{aligned}$$

If the value of  $T_a(x, y)$  is greater than zero then  $x$  and  $y$  are *not* independent (as supposed, in speaking of an *existent* relation between them); if  $T(\alpha, \beta) > 0$  then  $U_a$  and  $U_b$  are *not* independent. Conversely, if  $T_a(x, y) = 0$  or  $T(\alpha, \beta) = 0$  then, up to this moment in an observation, no dependency has been demonstrated. The *redundancy* is simply the degree of 'internal transmission' between possible universes regarded as 'the same' (for example, state variables or states of the same system) and is a measure of *constraint*.

$$\text{Redundancy} \triangleq T(x, y) = H^*(x, y) - H(x, y)$$

or if  $x, y$  are unique indices on 'the same' universe then

$$\begin{aligned} H^*_{\alpha} &= H_a(x) + H_a(y) \\ H_{\alpha} &= H_a(x, y) \end{aligned}$$

$$\text{Redundancy} \triangleq H^*_{\alpha} - H_{\alpha} = T(\alpha, x) \text{ (not } T(x, x) \text{ or } T(y, y)).$$

(1) Let  $i = 1, \dots, n$  be an index variable (that is, a quantity assuming one and only one value at once) running over a set of elements,  $s_i \in S$ , and pointing them out uniquely (there is a one to one correspondence between the values of  $i$  and  $n$  elements of  $S$ ).

Let  $p_1, \dots, p_n$  be probabilities so that  $1 \geq p_i \geq 0$  and so that  $\sum_i p_i = 1$ . The uncertainty/information measure over those elements of  $S$  indexed by  $i$  is

$$H(s) = -\sum_{i=1}^{i=n} p_i \log p_i; \quad \text{for } s_i \in S \quad (1)$$

which gives the *information*; the negative of this quantity is an *uncertainty*.

If it happens that a set  $S$  is a set of alternatives (for example, of initially exclusive and exhaustive events) then  $i$  runs over all (not just  $n$ ) elements.

(2) Let  $x$  and  $y$  be index variables and let  $p(x), p(y)$  stand for general designators of  $p$  values associated with their distinct values  $x_i, y_j; i = 1, \dots, n; j = 1, \dots, m$ . Using a shorthand due to Garner (1962).

$$H(x) = -\sum_i p(x) \log p(x) \equiv H(x) = -\sum_{i=1}^{i=n} p(x = x_i) \log p(x = x_i)$$

or

$$H(y) = -\sum_j p(y) \log p(y) \equiv H(y) = -\sum_{j=1}^{j=m} p(y = y_j) \log p(y = y_j)$$

These quantities are maximised if the  $p$  values are all equal and minimised if one is unity, and the others (by definition) zero. Thus in the case of  $x$  (for  $y$ , similarly) the maximum value, for  $n$  unchanging equally likely alternatives, is

$$H^*(x) = -\sum_n \frac{1}{n} \log \frac{1}{n} = \log n$$

Further  $H^*(x) \geq H(x) \geq 0$

It is certainly true that the values of  $x$  and  $y$  circumscribe a possible universe; it may be the universe  $U_a$ ,  $U_b$ ; in which case these quantities are identical with  $H(x)$  and  $H(y)$ . But this need not always be so; for example, if  $x$  and  $y$  are state variables of a system, then  $X$  (the set of values of  $x$ ) and  $Y$  (the set of values of  $y$ )

which is usually normalised to yield the specific measure

$$(3) \quad Z = \frac{H^*(\alpha) - H_\alpha}{H_\alpha} = 1 - \frac{H_\alpha^*}{H_\alpha}$$

(Until further notice, the subscripts  $\alpha, \beta$ , are omitted.)

(4) Conditional uncertainty/information/indices are derived from conditional probabilities of the form  $p(y|x = x_i)$  meaning 'the probability of each value of  $y$  (stated separately for all  $m$  values) if one specific value of  $x = x_i$  is given'. The selective uncertainty/information, given  $x_i = x$ , is

$$H(y|x = x_i) = - \sum_y p(y|x = x_i) \log p(y|x = x_i)$$

and the general conditional uncertainty/information is obtained as a weighted mean.

$$\begin{aligned} H(y|x) &= - \sum_x p(x) H(y|x = x_i) \\ &= - \sum_x p(x) \sum_y p(y|x) \log p(y|x) \end{aligned}$$

The converse term  $H(x|y)$  is obtained in the same manner and the following identities are demonstrable

$$\begin{aligned} H(x,y) &= H(x) + H(y|x) = H(y) + H(x|y) \\ T(x,y) &= T(y,x) = H(y) - H(y|x) = H(x) - H(x|y) \end{aligned}$$

For more than two variables (or universes) the total constraint is more complex. It is designated  $T^*(v,x,y)$  for variable  $v$ ,  $x$ ,  $y$  and is specified for any finite number of variables.

$$\begin{aligned} T^*(v,x,y) &= H^*(v,x,y) - H(v,x,y) \\ &= H(v) + H(x) + H(y) - H(v,x,y) \end{aligned}$$

The specific couplings (transmissions) are terms such as

$$T(v, \langle x,y \rangle)$$

representing the coupling between values of  $v$  and pairs of values  $\langle x,y \rangle$  of  $x$  and  $y$ . On decomposing the total constraint pairwise (recalling that  $T(x,y) = T(y,x)$ ) the following equalities are obtained:

$$\begin{aligned} T^*(v,x,y) &= T(v, \langle x,y \rangle) + T(x,y) \\ &= T(x, \langle v,y \rangle) + T(v,y) \\ &= T(y, \langle v,x \rangle) + T(v,x) \end{aligned}$$

so that, in summary

$$T^*(v,x,y) = T(v,x) + T(v,y) + T(x,y) + Q(v,x,y)$$

where the  $T$  terms are couplings between pairs of variables and  $Q(v,x,y)$  is an interaction (which may be positive, negative or zero) representing the trivariate

coupling, that cannot be expressed pair-wise (i.e. without considering the 'paired' coupling between 'one variable' and 'a pair of other variables').

$$(4) \quad Q(v,x,y) = T(v,y) + T(v,x) + T(x,y) - T^*(v,x,y)$$

For an  $m$  variable system, the expression has  $m$ -tuple,  $(m-1)$ -tuple, . . . terms. If the variables are indices on universes  $U_\alpha$ ,  $U_\beta$ ,  $U_\gamma$  that are held to be potentially distinct, then  $Q(v,x,y) = Q(\alpha,\beta,\gamma)$ .

(5) Von Foerster notes that redundancy ( $Z$  of equation 3) is one measure of organisation, and that a system is rightly regarded as self organising only if the rate of change of redundancy is positive (that is  $dZ/dt > 0$ ). Considering the statistical quantities  $H^*$  and  $H$  as variables in a statistical metalinguistic model of the system, differentiate the expression for  $Z$  with respect to time, thus obtaining

$$\frac{dZ}{dt} = - \frac{H^* \left( \frac{dH}{dt} \right) - H \left( \frac{dH^*}{dt} \right)}{H^{*2}}$$

For  $H^* > 0$  (that is, fully ordered systems are excluded) the condition that  $dZ/dt > 0$  is satisfied if and only if

$$H \frac{dH^*}{dt} > H^* \frac{dH}{dt}$$

of which there are two special cases (for one or other variable unchanging) namely

- (a) If  $\frac{dH^*}{dt} = 0$ , then  $0 > \frac{dH}{dt}$
- (b) If  $\frac{dH}{dt} = 0$ , then  $\frac{dH^*}{dt} > 0$

Case (a) corresponds to adaptation in a fixed universe. For example, in the universe of input/output behaviour ( $x$  indexing input,  $y$  indexing output) in a fixed field of contingencies or relations  $U_\alpha$ , where

$$\begin{aligned} H^*_{\alpha,v} &= H^*_{\alpha,x}(x) + H^*_{\alpha,y}(y) \\ H_\alpha &= H_\alpha(x,y) \end{aligned}$$

The rate of change is approximated by considering as distinct the universes  $U_\alpha(t)$ ,  $U_\alpha(t + \Delta t)$ . The self-organising condition is secured if, for any  $\Delta t$ , chosen the behavioural indices satisfy

$$H_{\alpha,t}(x,y) > H_{\alpha,t+\Delta t}(x,y)$$

Case (b) is of greater interest, since it entails the idea that  $U_\alpha$  does change,  $U_\alpha(t) \rightarrow U_\alpha(t + \Delta t)$  as, for example, by changing the field of attention. The required condition is secured if, for any value of  $\Delta t$ , the maximum uncertainty/information/index values (signifying possible behavioural relations) satisfy

$$H^*_{\alpha,t+\Delta t} > H^*_{\alpha,t}$$

## Appendix B

For example, this transformation of  $H^*$  may be performed by changing the learnable relations between  $x$  and  $y$ . In some cases but not all (see Chapter 5 onwards), it is permissible to express this operation as changing the structure of one universe  $U = U_\alpha \equiv U_\beta$  and thus transforming only  $H^*|_{t+\Delta t} > H^*|_t$ . This is so, for instance, if  $U_\alpha \subset U_\beta$ . Conversely, however, the transformation always can be expressed as an operation upon the universe:  $U_\alpha \rightarrow U_\beta$ . If  $H$  is to remain constant as required by condition (b) the transformation must usually be instantaneous (i.e. very rapid).

The general case is of greater interest and has two paradigms:

- For all  $\alpha, \beta, t, t + \Delta t: U_\alpha \subset U_\beta$  so that  $U = U_\beta$  and  $H^*$  is altered by reducing the constraint between  $x$  and  $y$  in  $U$ , as a result of which more relations can be learned. It is, as above, possible to express the same operation as a transformation  $\alpha \rightarrow \beta$ . Since there is usually no way of metrising the effect of relaxing constraint (through the sense of the change in  $H^*$  is determined) this operation is carried out in a steady-state system (see Chapter 7) until  $0 > dH/dt$  and if  $dH/dt$  approaches the value 0.
- For some  $\alpha, \beta, t, t + \Delta t, U_\alpha$  and  $U_\beta$  are not directly comparable, in the sense that an external observer's images of  $U_\alpha$  and  $U_\beta$  do not necessarily agree with the system's image. In this case (see Chapter 9 onwards) the transformation  $\alpha \rightarrow \beta$  that regulates  $H^*$  must be under the system's control though it may still be instigated by the same conditions on the rate of change of  $H$ .

Consider a test item with two alternative answers  $\{a, b\}$ . The student wishes to maximise his expected test score and the tester wishes to extract all available information about the student's present state of knowledge. Instead of requiring a simple 'yes—no' response to each alternative answer, the tester can seek more information about the student by asking him to assign probability numbers to the alternatives so as to represent his current state of doubt or degree of belief that a given alternative is the correct answer. In the case of two alternatives, the student is required to assign values  $P(a)$  and  $P(b)$  such that  $P(a) = 1 - P(b)$ . Clearly such responses are potentially more informative than simple 'yes—no' responses, since in the 'yes—no' case the student is restricted to only two possible response forms either  $P(a) = 1$  and  $P(b) = 0$  or  $P(a) = 0$  and  $P(b) = 0$ .

Given a set of probability numbers as responses, the tester has to compute a score. Assuming the probability numbers do represent a student's degree of belief, he may, for example, sum the values the student has assigned to the correct alternatives, perhaps also weighting items for their relative importance. This procedure is analogous to that of counting the number of correct responses in the 'yes—no' case, but suffers from a serious deficiency: the tester cannot validly assume the assigned probability numbers do represent the student's degree of belief, since a student who is a 'good statistician' will realise that when such a scoring scheme is employed, he can maximise his expected score by assigning probability numbers that do not represent his degree of belief. This is illustrated by the following example.

If the student's actual degree of belief that alternative 'a' is the correct answer is 0.75 and he follows instructions, setting  $P(a) = 0.75$  and  $P(b) = 0.25$ , he can expect a score of 0.75 with probability 0.75 and a score of 0.25 with probability 0.25. This gives an expected score of  $(0.75 \times 0.75) + (0.25 \times 0.25) = 0.625$ . However, the student can make his expected score larger by assigning a larger probability number to alternative 'a'. In fact, he maximises his expected score by assigning  $P(a) = 1.00$ . This gives an expected score of  $(0.75 \times 1.00) + (0.25 \times 0.00) = 0.75$ . In general, the 'good statistician' can maximise his expected score by assigning a value of 1.00 if his degree of belief that an alternative is correct is greater than 0.50, and by assigning a value of 0.00 if his degree of belief that an alternative is incorrect is less than 0.50. If his degree of belief is 0.50, it makes no difference what value he assigns since all values yield equal expected scores.

What is required is a scoring system where the student's optimal strategy is to

assign probability numbers that do reflect his degree of belief. Shuford, Massengill and Albert (1966) derive the necessary and sufficient conditions for a scoring system that satisfies this criterion for the case of two alternatives and show how to create a virtually inexhaustible number of such systems. In addition, they give partial results for the case of more than two alternatives.

An example scoring system for the case of two alternatives is one which specifies:

- (1) that the student receives a score of  $1 - (1 - P(a))^2$  points if he assigns  $P(a)$  as his degree of belief that alternative 'a' is correct when it is in fact correct, and
- (2) that he receives  $1 - (P(a))^2$  if he assigns  $P(a)$  to alternative 'a' when it is, in fact, incorrect.

Thus, if as before, the student has a degree of belief of 0.75 that alternative 'a' is correct and assigns  $P(a) = 0.75$ , his expected score is  $0.75 \times (1 - (1 - 0.75)^2) + 0.25(1 - (1 - 0.25)^2) = 0.811$ . Assigning  $P(a)$  less than or greater than 0.75 leads to a lower expected score.

The following scheme (mentioned in the text) can be employed for the case of more than two alternatives and has the advantage over other such schemes that the student's score depends only on the value he assigns to the correct alternative.

Given the correct answer from a set of exclusive alternatives is the  $i$ th alternative and the student's assigned value to the  $i$ th alternative is  $P_i$ , this scheme specifies that:

- (1) if  $P_i$  is greater than or equal to 0.1, he receives  $1 + \log P_i$  points, and
- (2) if  $P_i$  is less than 0.1 he receives  $-1$  points.

Condition (2) provides a cut off point that gives a point scale running from  $-1$  to  $+1$  (rather than  $-\infty$  to  $+1$ ). Thus, for extreme values of  $P_i$ , some information is lost. In any practical applications, this loss in accuracy is insignificant.

## Appendix C

### 1 Fuzzy Operations (Zadeh)

For a thorough description the reader is referred to Goguen (1968) and Zadeh (1973). The following notes and definitions are chiefly intended to indicate the kind of operation we have in mind and to convince the diffident reader that the concept of a fuzzy operation is not, itself, fuzzy (or vague).

A fuzzy subset  $X$  of a universe  $U$  (here a set of objects, see footnote on the first page of Chapter 1) is defined by a function  $C_X(u)$  or  $C_X: U \rightarrow \{ \text{Interval } 1, 0 \}$  that assigns a 'grade of membership' to each element of  $U$ . Those elements of  $U$  that are associated with positive values ( $C$ ) of  $C_X(u)$  are called the *support* of  $X$ . A fuzzy unit set  $\{x\}$  has one support  $u \in U$  and a positive grade of membership  $C_{\{x\}}(U)$ . Using a slash notation to designate grades of membership, i.e.  $C/u$ . A non-fuzzy unit set is  $1/u$ .  $X$  itself is the union (written UN, Zadeh uses the integral to comprehend indefinitely large unions) of certain fuzzy unit sets. That is

$$X \triangleq \frac{\text{UN}(C_x(u)/u)}{u \in \text{support}}$$

The complement of  $X$  is

$$\text{Not } X \triangleq \frac{\text{UN}/(1 - C(u)/u)}{u \in U}$$

The union of fuzzy sets  $X$ ,  $Y$ , is

$$\text{Or } (X, Y) \triangleq \frac{(\text{UN})[C_X(u) \vee C_Y(u)]/u}{u \in U}$$

where  $\vee$  is the maximal operator;

$$r \vee s = \text{Max } (r, s) \triangleq \begin{cases} r & \text{if } r \geq s \\ s & \text{if } s > r \end{cases}$$

The intersection of fuzzy sets  $X$ ,  $Y$  is

$$\text{and } (X, Y) \triangleq \frac{\text{UN}[C_X(u) \wedge C_Y(u)]/u}{u \in U}$$

where  $\wedge$  is the minimal operator

$$r \wedge s = \text{Min } (r, s) \triangleq \begin{cases} r & \text{if } s > r \\ s & \text{if } r \geq s \end{cases}$$

A fuzzy relation,  $R$ , is a fuzzy subset of a cartesian product; if  $R$  relates  $X$  to  $Y$  then

$$R \triangleq \frac{UN}{x \times y} C_n(x,y) / \langle x,y \rangle : x \in X, y \in Y.$$

If  $R$  and  $S$  are fuzzy relations between  $X$ ,  $Y$  and  $Z$ , their composition is

$$R \circ S \triangleq \frac{UN}{x \times z} Supremum (C_n(x,y) \triangle C_n(y,z)) / \langle x,z \rangle$$

where 'supremum' is used in the usual sense of 'collection of maximal elements' of a relation and  $C_n(x,y)$  is a grade of membership function of two or more variables. One special case of a fuzzy relation is the fuzzy cartesian product of universes  $U$ ,  $V$  with elements  $u$ ,  $v$  namely,

$$X \otimes Y \triangleq \frac{UN}{U \times V} c_X(u) \triangle C_Y(v) / \langle u,v \rangle$$

Any non fuzzy (standard) conditional: 'If  $x$ , then  $\beta$  else  $\eta$ ' is equivalent (since 'if . . . then' is an implication  $\rightarrow$ ), to the Boolean expression  $(\alpha \rightarrow \beta) \wedge (\alpha^c \rightarrow \eta)$  where the ' $\sim$ ' stands for Boolean negation. By extension, the *fuzzy conditional* is defined (for the same or different universes,  $U$ ,  $V$  and fuzzy sets  $W$ ,  $X$ ,  $Y$ ), as

$$\text{'If } X \text{ then } Y \text{ else } W' \triangleq X \otimes Y \text{ or } (\text{not } (x) \otimes W)$$

which expresses, in executable or intensional form, a fuzzy relation,  $R$ , from  $U$  to  $V$  and which is generalised to image the (standard) extended conditional form

$$\text{'If } X_1 \text{ then } Y_1 \text{ else if } X_2 \text{ then } Y_2 \text{ else } \dots \text{ if } X_n \text{ then } Y_n'.$$

A *fuzzy algorithm* is a non-fuzzily indexed (i.e. well ordered) sequence of instructions of this form (or the degenerate forms' unconditional imperative') and 'assignments' using, in execution, the major inference rule (for a fuzzy relation  $R$  from domain  $X$  to codomain  $Y$ ) which is

$$Y = X \circ R$$

The execution is *numerical* if each supremum is resolved by ranking (using any consistent ranking rule) to select a unique outcome.

## 2 Finite-Function Machines (Von Foerster)

These comments establish the salient distinction between finite-state machines and finite-function machines; the serious reader is (as usual) referred to the original papers.

The characterising equations of a finite-state machine are

$$u_{\tau+1} = f(u_\tau, z_\tau)$$

$$v_\tau = g(u_\tau, z_\tau)$$

and may be expanded to eliminate all but a finite history of  $n$  inputs and an initial internal state or output state. By this construction, it is possible to avoid direct

reference to any internal state function,  $f$ . Thus, supposing  $g$  has an inverse  $z = \phi(u,v)$ , it follows that

$$v_{\tau+1} = g(u_{\tau+1}, f(u_\tau, \phi(v_\tau, u_\tau)))$$

or that

$$v_\tau = g^{-1}(u_\tau, u_{\tau+1}, v_{\tau+1})$$

but (from the last equation)

$$v_{\tau+1} = g^1(u_{\tau+1}, u_{\tau-2}, v_{\tau-2});$$

which, if substituted in the last equation, gives

$$v_\tau = g^{\sigma 2}(u_\tau, u_{\tau-1}, u_{\tau-2}, v_{\tau-2})$$

or, for  $n$  stages

$$v_\tau = g^n(u_\tau, u_{\tau-1}, \dots, u_{\tau-n}, v_{\tau-n})$$

which is a *recursive function* (the behavioural history) containing only one reference to an operation or computation: namely  $\langle u_{\tau-n}, v_{\tau-n} \rangle$  (or, equivalently, the internal state at  $\tau - n$ : namely  $z_{\tau-n} = \phi(u_{\tau-n}, v_{\tau-n})$ ).

Turning to a finite-function machine, its characterising equations are

$$f_{\tau+1} = F(u_\tau, f_\tau)$$

$$g_\tau = G(u_\tau, f_\tau)$$

and are thus isomorphic with the finite-state machine equations under the correspondences given below.

$$\begin{aligned} \text{Next internal state function} &\longleftrightarrow \text{Internal state} \\ \text{Output state function} &\longleftrightarrow \text{Output state} \end{aligned}$$

If we apply the steps which lead to the expression for a finite-state machine, stripped of direct reference to internal states (in terms of a recursive function  $g^n$ ), we obtain

$$g = G^n(u_\tau, u_{\tau-1}, \dots, u_{\tau-n}), (g_{\tau-1}, g_{\tau-2}, \dots, g_{\tau-n})$$

which is a *recursive functional*.

The important difference is that the recursive functional contains explicit reference to a history of operations (the several  $g_{\tau-n}$ ) whereas the recursive function does not (just one  $v_{\tau-n}$ ). This is the salient distinction between a finite-state machine and a finite-function machine. Only the latter contains the computations needed to reconstruct an image of its history.

## Appendix D

### Appendices

**Specification 1 Model I Simulation**

*Nodes in the environment.* Sum = 256. Nodes are indexed by a number  $k$  or by coordinates  $\alpha, \beta$ , equivalent to  $k$  (as in Fig. 22). Each node designates a food store. *Food store.* The amount of food at node  $k$  at trial  $n$  is  $u_k(n)$ , maximum value 128. *Food increment.* For  $u_k(n)$  less than 128, the amount of food added at trial  $n$  is  $\Delta u_k(n)$ , which is a function of  $O_k(n)$ , the average occupancy of node  $k$  over the previous 8 trials and a parameter  $c$  (with values 1, 2, or 4 determined by the experimenter). This function is:  $\Delta u_k(n) = c$ , if  $O_k(n) = 0$  or 1;  $\Delta u_k(n) = c(1 + O_k(n))$ , if  $O_k(n) > 1$ .

*Occupancy.* The number of automata at node  $k$  at trial  $n$  is  $\text{Occupancy}_k(n)$ . The average value of  $\text{Occupancy}_k(n)$  over the previous 8 trials is  $O_k(n)$ .

*Motions of an automaton indexed  $i$ .* Consider automaton  $i$  at node  $k = \alpha, \beta$ , at trial  $n$ . As in Fig. 22, it can remain where it is or move into any of the positions  $\alpha - 1, \beta + 1; \alpha, \beta + 1; \alpha + 1, \beta - 1; \alpha, \beta - 1; \alpha + 1, \beta - 1$ ; provided that it is not at the edge of the field. These motions are indexed with a number,  $\rho$ , between 0 and 8 (0 designating the act of remaining where it is).

*Sensory capability.* Automaton  $i$  senses the value of  $u$  at any node to which it can move. Hence it has 9 sensory inputs indexed by  $\rho$ .

*Stomach content.* The content of the stomach of automaton  $i$  at the start of trial  $n$  is  $\theta_i(n)$ , maximum value 128. If  $i$  is at node  $k$ , then  $\theta_i(n)$  is incremented by  $\Delta \theta_i(n) = \eta \cdot u_k(n) - \theta_i(n)$  if  $u_k(n) > \theta_i(n)$ . If  $\theta_i(n) > u_k(n)$ , the value of  $\theta_i(n)$  is unchanged.  $1 > \eta > 0$ .

*Age of  $i$ .* The age of an automaton is the number of trials since it entered the simulation.

*Aging and metabolism.* For each trial the value of  $\theta_i(n)$  is decremented by an amount  $F(\text{age of } i)$ . This function has the following form (though several functions were used in the experiments):

$F(\text{age of } i)$	1	1	1	1	1	1	2	2	4	4	8	8	16	16	32	64		
$\text{Age of } i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$> 16$	$> 24$

*Decay.* Automaton  $i$  is deleted from the simulation at trial  $n$  if  $0 > \theta_i(n)$ .

**Specification 2 Model II Simulation**

*Nodes in the environment.* Number and connectivity as in Specification 1.

*Food store and incrementation.*

$$u_k(n) = C \cdot \text{Occupancy}_k(n - 4) + C \cdot \text{Occupancy}_k(n - 3) + C \cdot \text{Occupancy}_k(n - 2) + C \cdot \text{Occupancy}_k(n - 1).$$

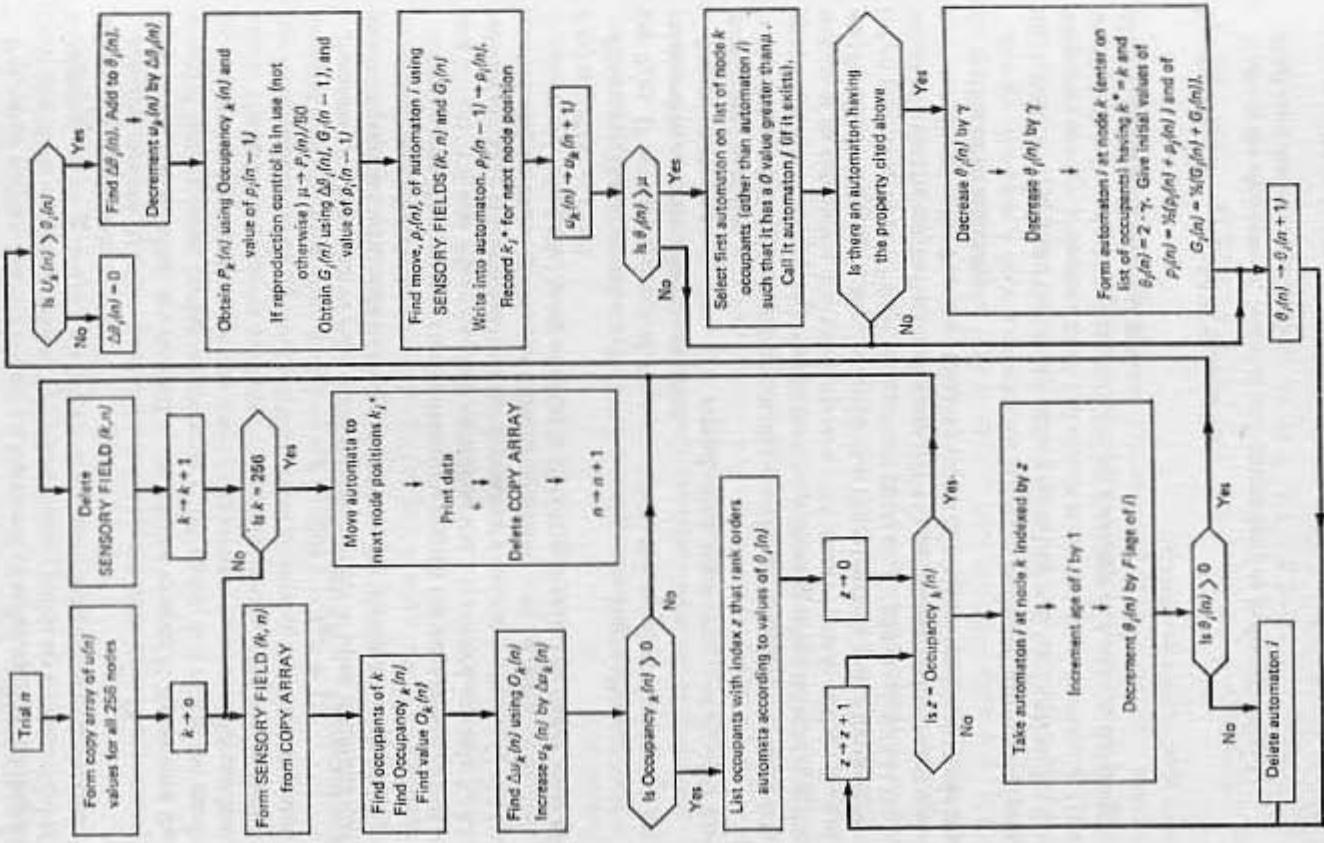


Figure 107 Flow-chart for Model I Simulation.

The automata do not 'eat' separately. The available food is equally divided between the occupants of a node and an automaton receives the YIELD of a node.

*Occupancy.* As in Specification 1; the average occupancy, however, is computed over only 4 trials. (For details, consult the flow chart of Figs. 107 and 108.)

*Types of automata.* There are 4 types of automata, denoted  $-|$ ,  $+$ ,  $\times$ , and their possible motions are shown in Fig. 22.

*Sensory capabilities.* An automaton can sense the value of  $u$  at any node to which it can move. Since different 'types' of automata have different permitted motions, they also have different sensory fields.

*Stomach content.* Similar to Specification 1. As in Specification 1, however, the value of  $\Delta\theta_i(n)$  is the same for all automata residing at node  $k$  at trial  $n$ . This value is the amount of food at node  $k$  divided by the number of automata residing at this node.

*Age of i.* As in Specification 1.

*Aging and metabolism.* As in Specification 1, except that the function  $F(\text{age of } i)$  can be differently defined for each type of automaton.

*Decay.* As in Specification 1.

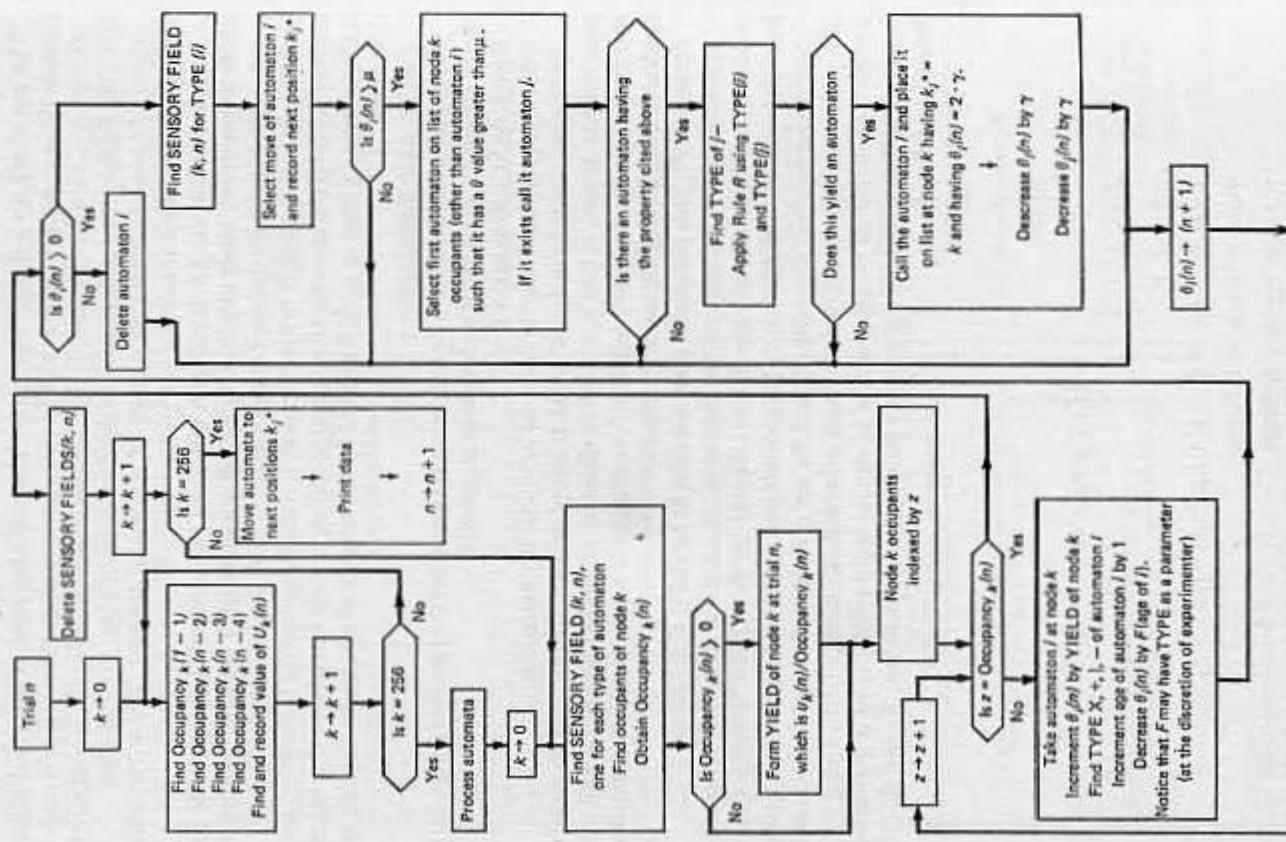
*Experience weights.* These do not exist in Model II simulations.

*Selecting motions.* As in Specification 1, except that automaton  $i$  has no experience weights. Hence, at trial  $n$ , it selects a direction of motion associated with the maximum among its sensory inputs (if no maximum exists, it selects a motion randomly from among the maximal subsets).

*Parameter  $P$ .* This parameter does not exist in Model II simulations.

*Reproductions.* The conditions for reproduction are those described in Specification 1, but the results of reproduction depend on the types of automata that encounter one another and satisfy these conditions at the trial concerned. A pair of alternative reproduction rules,  $R_1$  and  $R_2$ , are defined, and one of these is used in a particular experiment (unless stated otherwise,  $R_1$  is employed). Using  $\sigma$  to denote a combination of automata,  $p$  to denote the probability of a particular outcome, and  $\Rightarrow$  to denote the progeny produced as an outcome,  $R_1$  and  $R_2$  are defined as follows.

$R_1 = -o-$	$ $
$ o  \Rightarrow  $	$-o  =  o- \Rightarrow p = \mathbb{I}(+), p = \mathbb{I}(X)$
$+o+$	$\Rightarrow +$
$\times o \times$	$\Rightarrow \times$
$+o-$	$= -o+ \Rightarrow p = \mathbb{I}(+), p = \mathbb{I}(X)$
$\times o-$	$= -o \times \Rightarrow \text{Infertile}$
$+o  =  o+$	$\Rightarrow \text{Infertile}$
$\times o  =  o \times$	$\Rightarrow \text{Infertile}$



$R_2 \Rightarrow -o- \Rightarrow -$   
 $|o| \Rightarrow |$   
 $-o| = |o- \Rightarrow p(+)=\frac{1}{2}, p(x)=\frac{1}{2}$   
 $+o+ \Rightarrow p(+)=\frac{1}{2}, p(\text{Infertile})=\frac{1}{2}$   
 $x o x \Rightarrow p(x)=\frac{1}{2}, p(\text{Infertile})=\frac{1}{2}$   
 $+o x = x o+ \Rightarrow p=\frac{1}{2}(+),$   
 $p(x)=\frac{1}{2}, p(\text{Infertile})=\frac{1}{2}$   
 $+o- = -o+ \Rightarrow \text{Infertile}$   
 $x o- = -o x \Rightarrow \text{Infertile}$   
 $+o| = |o+ \Rightarrow \text{Infertile}$   
 $x o| = |o x \Rightarrow \text{Infertile}.$

*Inheritance.* Apart from the type determination entailed by  $R_1$  and  $R_2$ , there is no carryover from a given generation to the next generation. Indeed, there are no properties  $G$  or  $P$  to inherit.

*Processing.* The automata are processed in an arbitrarily selected order. *Parallel simulation.* Since  $u_k(n)$  is not modified in the course of processing node  $k$ , the COPY ARRAY is unnecessary and the current values of  $u(n)$  are computed by the program at the beginning of a trial. SENSORY FIELD is derived from this collection of values, only there are 4 different SENSORY FIELDS, one for each type of automaton, namely  $x, +, |, -$ .

*Storage allocation.* A node is a set of  $k$ -indexed words specifying  $u_k(n)$  and the 4 previous occupancies used in computing the value of  $u_k(n)$ . An automaton is a set of  $k$ -indexed words that specify  $\theta_k(n)$ , age of  $i$ , and the location, at trial  $n$ , of the automaton.

Figure 108 Flow-chart for Model II Simulation.

## Glossary

The items are arranged in an order conducive to rational development. The list is short enough to be scanned readily and the looked-for-item discovered without difficulty.

**Set S** A collection of unordered unitary elements. Membership in the collection is written ' $\in$ ' so that  $s \in S$  means 'element  $s$  is a member of set  $S$ '.

**Subset X** a 'portion of a set' (written,  $X \subset S$ ) 'either a portion of or all of, a set' (written,  $X \subseteq S$ ). If the elements of subsets are specified explicitly the elements in question are enclosed in brackets (curly brackets, to distinguish them from *ordered sets* as below). For example, if  $S = \{a,b,c\}$  then some subsets are  $X = \{a,b\}$  and  $Y = \{a,c\}$ .

**Power Set** The set of all subsets of a set (usually, including the *null* or *empty* set). For example, if  $S$  has elements  $\{a,b,c\}$  then the Power set of  $S$  is  $\{\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \text{Null}\}$ .

**Intersection, Union** If  $X$  and  $Y$  are subsets of  $S$  then  $X \cap Y$  is the *intersection* of  $X$  and  $Y$  (the subset of elements of  $S$  that belong to  $X$  and to  $Y$ ) whilst  $X \cup Y$  is the *union* of  $X$  with  $Y$  (the subset of elements of  $S$  belonging either to  $X$  or to  $Y$  or both).

**Complement with respect to S** The complement of  $X$  is  $S - X$ ; the elements of  $S$  that do not belong to  $X$ .

**Disjoint Subsets of S** are subsets,  $X, Y (X \subset S, Y \subset S)$  such that  $X \cap Y$  is void. Hence,  $X \subseteq S = Y$  and  $Y \subseteq S = X$ .

**Property** That which is common to the members of a set (for example, if  $S$  is the set of things,  $X$  of red things, then  $X$  is the property 'redness').

**Cartesian Product** for two or more sets,  $S$  and  $T$ , their Cartesian product,  $S \times T$ , is the collection of all ordered pairs  $\langle s,t \rangle$  with one element taken from  $S$ , and one from  $T$  (for example, all pairs of men and women).

**Ordered pairs** are often called 2-tuples; this generalises, for an *ordered set of n elements*, to  $n$ -tuples  $\langle s_1 \dots s_n \rangle \in S_1 \times \dots \times S_n$ .

**Relation R** A relation, given as a listing, is a subset  $R \subseteq S \times T$  (for example, if  $R$  is 'married to', then  $R$  contains all those men and women who are husband and wife).  $R$  may also be represented by a mapping or series of linkages between the elements of  $S$  and  $T$ . In a monogamous society  $R$  is a one to one mapping. Generally,  $R$  may consist in any one, one; one, many; many, one; or many, many mapping and it may involve all or only some of the elements (those involved are called  $R$ 's *domain* and *codomain* or *range*). In general, also  $R \subseteq S_1 \times \dots \times S_n$ , and  $n$  is called the adicity of the relation (thus  $R \subseteq S \times T$  is a 2-adic or binary relation).

**Function F** A kind of relation; either a one to one or many to one mapping with specified domain. If the domain is less than an (understood) set the function is a 'partial' function. Since  $F$  is specified as a relation  $F \subseteq S_1 \times \dots \times S_n$ ,  $n$  is the adicity of the function (or, equivalently,  $F$  is a function of  $n$  variables).

**Variable** A named indexing device that assumes but one value at once. For example, a variable  $u$  that ranges over the elements  $s$  of  $S$ ; a variable  $v$  that ranges over the elements  $t$  of  $T$ . The values of  $u$  denote elements of  $S$ , the values of  $v$  denote elements of  $T$ ; in general, a vector  $\langle u_1 \dots u_n \rangle$  of  $n$  variables index the elements of  $S_1 \times \dots \times S_n$  and its values stand for  $n$ -tuples  $\langle s_1 \dots s_n \rangle \in S_1 \times \dots \times S_n$ .

**Classes of Functions** Functions are often classified by their domain/range for example, 'Boolean Functions' of a variable assuming values {0,1} (or 'True/False') and having possible values {0,1}; an integer-arithmetic function (of a variable with integer values). Functions are also classified according to the number of variables needed to index their domain. Thus  $F(u)$  is a 1 place function.

**Assertoric Logic** (In contrast to command logics and question logics considered in the text). A language together with rules for usage (syntax) and inference (see below). Statements in the language are given semantic interpretations; commonly as denoting relations and conditions that hold in one or more sets.

**Proposition p** (in a language) A statement like 's is red' or 's is married to t' and interpreted, if true, as ' $s \in X$ ' or ' $\langle s,t \rangle \in R \subseteq S \times T$ '.

**Predicate P** An adjective. Equisignificantly, the name for a (usually partial) function from a set  $S$  to a set of truth values. Commonly, the truth value set is taken to be {1,0} (indicating 'true' and 'false') but this is not always the case (see, for example, the Fuzzy sets in Appendix C). If  $u$  is a variable indicating members,  $s$ , of a set,  $S$ , then  $P(u) = \text{True}$ , is a *one place* predicate statement, meaning that the pointed at element has the *property*

**named by  $P$ .** If  $P$  is used to name  $X \subset S$  then  $X$  is (as before) a property and  $P(u)$  is true for all elements  $S$  (pointed at by  $u$ ) that are members of  $X$ . A 2-place predicate  $Q(u,v)$  stands for a 2-adic relation.  $Q(u,v)$  is true when  $u$  and  $v$  point at elements,  $\langle s_0, t_0 \rangle$ , of  $S$  and  $T$  such that  $\langle s_0, t_0 \rangle$  is a married couple). In general, there are  $n$ -place predicates  $Q(u_1, \dots, u_n)$ .

**Conjunction** For propositions  $p,q$  written ' $p \wedge q$ ' meaning that ' $p$  and  $q$ ' is true if  $p$  and  $q$  are both true.

**Disjunction** (unqualified and inclusive) written ' $p \vee q$ ' meaning that ' $p \vee q$ ' is true if either  $p$  or  $q$  or both are true.

**Negation** The proposition  $\text{Not-}p$ . If  $p$  asserts that  $s \in X$  then  $\text{Not-}p$  asserts that  $s \in S - X$ , the complement of  $X$  in  $S$ .

**Exclusive Disjunction** For propositions,  $p, q, r$ , the exclusive disjunction written ' $p \vee q \vee r$ ' means, if true, that one and only one of  $p, q, r$  is true. Consequently, exclusive disjunctions are often known as alternative sets, and are used for example, to represent alternative answers to a question.

**Implication**  $p \rightarrow q$ , means that if  $p$  is true, then  $q$  is true.

**Equivalence**  $p \rightarrow q$  and  $q \rightarrow p$ ; written  $p \equiv q$ . For example,  $p \rightarrow q \equiv \text{Not } p \vee q$ .

**Inference** Typical inference rules are the primary rule (Modus Ponens of classical logic); if  $p \rightarrow q$  and ' $p$ ' is true then it is legitimate to infer that ' $q$ ' is true; and Modus Tollens (If ' $p \rightarrow q$ ' is true and  $q$  is false then it is permissible to infer the falsity of  $p$ ). Inference Rules figure as metastatements about the language employed to accommodate the logical system and are necessarily augmented by other (somewhat obvious) metatheoretical manipulative permissions; for example, substitution and replacement and logical truism or tautology (if  $q$  is derived by substitution and replacement from  $p$  then the statement  $p \rightarrow q$  is a tautology).

**If then else conditional imperative** 'If  $A$  then  $B$ , else  $C$ ' (where  $A, B, C$  are expressions in an action or programming language) means, when the label attached to this statement is encountered:

'If condition  $A$  is satisfied, bring about (do, secure or evaluate)  $B$ ; if not, bring about  $C$  (and, having done so, proceed to the next label).

The execution of a conditional imperative satisfies or brings about a function or a relation (it invariably does so, though other effects may be more obtrusive). So does the execution of an algorithm or program.

**Assignment Statement** (written  $u \Rightarrow v$  or  $u \rightarrow 5$ ) meaning 'give  $u$  the value of  $v$ ' or 'give  $u$  the value of  $5$ '.

**Program** An (ordered) set of assignment statements, and conditional imperative statements, which is open to execution, as a process with a well specified beginning and ending.

**Recursive Definition** A non viciously circular definition, such as the following definition of 'oddments'.

**Base**  $X$  is an oddment;  $Y$  is an oddment.

**Recursive Part** An oddment placed on an oddment, is an oddment. For example,  $X$  on  $Y$ , or  $X$  on ( $X$  on  $Y$ ) or  $Y$  on  $X$  on ... ( $X$  on  $Y$ ).

**Exclusion or uniqueness Requirement** Nothing other than these is an oddment.

It is assumed that whoever, or whatever, employs this definition is able to recognise  $X$ ,  $Y$  and to perform the 'operation' on and to recognise the result of applying on.

**Recursively or constructively specified functions**

**Base** Recursive Functions are simple functions, namely, Constant valued functions (like  $f(x) = \text{Constant}$ , for all values of  $x$ ) or simple order functions (like the successor function,  $f(x) = x + 1$  and functions that search an index or move an object in a constant manner; for example  $f(x) = (\text{next above } x)$  or Identity Functions mapping a (possibly complex) element into a property or itself; thus  $f(x) = x$ , or  $f(\langle a,b,c \rangle) = b$ .

**Recursive Part** If composition, written ' $o$ ', means doing one thing after another then any composition of simple functions is a recursive function; for example,  $f \circ g$ , or  $f \circ f \circ g$ .

Any class of functions of which the member functions are orderable (by a simple order function) and are generated from some ancestor function by a common operation, is a recursive function.

Any functions of  $m$  variables generated by substitution, with constant value assignment, from a recursive function of  $m + 1$  variables is a recursive function.

**Exclusion Rule** No other functions are recursive functions.

**Compiler** A program for translating programs, expressed conveniently in a 'high level' and humanly understandable (but, all the same 'formal') language such as ALGOL, or FORTRAN or PLANNER, into a machine code that is interpreted at a mechanical level (and is fussy enough to defeat human patience). No detailed knowledge of high level or machine languages is required; the interested reader may refer to B. Higman's *A Comparative Study of Programming Languages* (MacDonald, 1967).

**Miscellaneous Mathematical terms** A graph is a structure consisting of *nodes* (drawn as points) and connecting *arcs* that join some or all of the nodes. In a *directed graph* the arcs are directional and often called *edges*. Both nodes and edges are labelled (for example, nodes may represent members of a group; edges, paths of communication between the members). Formally a directed graph is a binary relation on a set of nodes. Though graphs are often invoked, familiarity with them is not essential. The interested reader should consult C. Berge's *The Theory of Graphs* (Methuen, 1962) and Busaker and Saaty's *Finite Graphs and Networks* (McGraw-Hill, 1965). Occasionally we refer to *Games* with the meaning given in *game theory* or *Decision Theory*. Again, little background knowledge is necessary, but readers intrigued by Game Theory will find Luce and Raiffa's *Games and Decisions* (Wiley, 1957) still the most readable and informative reference.

**Miscellaneous Statistical Terms Used in discussing experimental results** The following comments are superficial but easily augmented by reference to any elementary statistics text book. The *mean* of a set of observations is their average (arithmetic mean) value. The *variance* is an index of scatter about the mean (usually the *standard deviation*, abbreviated as SD is cited; this quantity is the square root of the variance). A significance level (cited as a probability or a percentage) is attached to any serious comment or finding; the usual values are 1% and 0.1% (or  $p = 0.01$  and  $p = 0.001$ ). Thus an assertion that any numerical quantity is significant at the 0.1% level (or  $0.001 \geq p$ ) means that the quantity might have assumed this value haphazardly or randomly with 1 chance in 1000. Statistical tests are employed in stating hypotheses about absolute values, *differences* between mean values, and *trends* (for example, 'the difference is significant at the 0.1% level'). *Correlation* coefficients measure the undirected (other than random) association between quantities; once again, correlations are quoted together with a properly calculated level of significance.

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- The index is selective and (deliberately) incomplete. Certain words or phrases appear often and are omitted; for example, 'Learning', 'Adaptation' (as psychological omissions); 'Feedback', 'System', 'Goal Directed', 'Alternative Set', 'Decision' (as Cybernetic or System Theoretic omissions); and 'Inference', 'Deduction/Induction' (in the field of logic and philosophy). The remaining, less common, words are usually indexed to points of discussion, rather than places where they are mentioned in the text.
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